Fifth-order corrected field descriptions of the Hermite-Gaussian (0,0) and (0,1) mode laser beam

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In this paper, we extend the work of Barton and Alexander [J. Appl. Phys. **66**, 2800 (1989)] on the fifth-order corrected field expressions for a Hermite-Gaussian (0,0) mode laser beam to more general cases with adjustable parameters. The parametric dependence of the electron dynamics is investigated by numerical methods. Finally, the fifth-order corrected field equations for the Hermite-Gaussian (0,1) mode are also presented.

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I. INTRODUCTION

During the past two decades, considerable efforts have been made to investigate the solutions to Maxwell's equations in vacuum [1–4], which are of crucial importance in the study of laser-matter interactions, such as electron acceleration by far-field intense lasers [5–7] and localized transmission of electromagnetic energy [8–10]. The initial progress in this direction was the discovery of a small longitudinal field component of electromagnetic radiation with finite transverse spatial extension [1]. Later it was found that only by utilizing the field equations including all components [4–11] can we explain the well-known experimental results of Borham and co-workers [12], in which they observed the independence of the energy of the emitted electrons of the polarization of the laser fields. Another example showing the necessity of accurate field descriptions can be found in the intensive discussions [13] resulting from the theoretical model used by Malka *et al.* [14] in their laser acceleration experiment, in which electrons of 1 MeV energies were observed. The most discussed point in their model is the neglect of the longitudinal field component. Recently, we presented an interpretation corresponding to their experimental results with the full field components in Ref. [7].

Generally speaking, there are three categories of field description. One is the exact analytical solution of Maxwell's equations, for example ranging from the simple plane wave solution and nondiffracted Bessel beams [3] to EDEPT(electromagnetic directed-energy pulse train) solutions [8], splash modes [15], EM missiles [16], EM bullets [17] and transient beams [18]. At present most of these fields have been discussed mainly in theoretical work since there are a lot of difficulties in producing them in experiments. The second category is the exact integral solutions using the angular spectrum method [5,6], i.e., building the field equations from the superposition of plane waves based upon certain boundary conditions. The problem with this group of solutions lies in their intensive consumption of computer time, which makes comprehensive numerical study and analysis very difficult and inefficient. The third category are the approximate field equations [1], which are summations of factors to different orders of $s=1/kw_0$, where k and w_0 are the wave number and waist radius of the laser beam, respectively. Until now, this group of field descriptions, mostly Hermite-Gaussian beams, has been widely used in laser acceleration studies [19] because of its relatively simple analytical form. To describe tightly focused beams, where w_0 is nearly of the same order as the wavelength, Barton and Alexander [20] modified the paraxial approximation introduced by Davis [21] to fifth-order corrected equations. In this paper, we will go a step forward by extending their fifth-order corrected field equations to more general cases with two adjustable parameters. Every set of fixed values of the parameters can provide us with one group of fifth-order corrected field equations. This may give us more opportunities to study the electron behavior with different field descriptions and to explore the characteristics of the electron dynamics. Furthermore, the fifth-order corrected field equations for Hermite-Gaussian (0,1) mode laser beams are also obtained. These results should be helpful to anyone needing to work with the full field descriptions of a stationary laser beam, which satisfy the Maxwell's equations accurately.

In the following, Sec. II is devoted to the theoretical development of the field equations of the Hermite-Gaussian (0,0) mode laser beam. In Sec. III, we present some numerical examples to demonstrate the parametric dependence of the electron dynamics in intense lasers. In Sec. IV, following the same steps as in Sec. II, we obtain the fifth-order corrected field equations for the Hermite-Gaussian (0,1) mode. The final part is a summary.

II. THEORETICAL DEVELOPMENT FOR THE HERMITE-GAUSSIAN (0,0) MODE LASER BEAM

For a stationary laser beam propagating in vacuum, the harmonic time dependence is assumed to be $e^{i\omega t}$. For convenience in the following discussions, we will drop all the time-dependence terms in the subsequent formulas. Then the Maxwell equations take the forms

$$\vec{\nabla}\cdot\vec{E}=0, \tag{1}$$

$$\vec{\nabla}\times\vec{E}+i\omega\vec{B}=\vec{0}, \tag{2}$$

$$\vec{\nabla}\cdot\vec{B}=0, \tag{3}$$

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$$\vec{\nabla}\times\vec{B}-\frac{i\omega}{c^2}\vec{E}=\vec{0}. \tag{4}$$

Throughout this paper, SI units are used. Solutions to the above equations can be found by constructing a Hertz vector oriented, for example, along a transverse direction,

$$\vec{M}=\psi(x,y,z)e^{-ikz}\hat{e}_x, \tag{5}$$

which satisfies the Helmholtz equation, i.e.,

$$\vec{\nabla}^2\vec{M}+k^2\vec{M}=\vec{0}. \tag{6}$$

We will work under the Lorentz gauge as in Ref. [20] with $\vec{A}=\vec{M}$ and $\phi=(ic/k)\vec{\nabla}\cdot\vec{M}$. Then

$$\vec{E}=-i\omega\vec{M}-\frac{ic}{k}\vec{\nabla}\vec{\nabla}\vec{M}, \tag{7}$$

$$\vec{B}=\vec{\nabla}\times\vec{M}. \tag{8}$$

To obtain the Hertz vector, we substitute Eq. (5) into Eq. (6). Then

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-2ik\frac{\partial}{\partial z}\right)\psi=0. \tag{9}$$

As usual [20,21], we normalize x,y by the beam width w_0 and z by the diffraction length kw_0^2, i.e.,

$$\xi=\frac{x}{w_0},\qquad \eta=\frac{y}{w_0},\qquad \zeta=\frac{z}{kw_0^2}. \tag{10}$$

So the Helmholtz equation can be rearranged as

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}-2i\frac{\partial}{\partial\zeta}\right)\psi=-s^2\frac{\partial^2}{\partial\zeta^2}\psi \tag{11}$$

with $s=1/kw_0^2$. If s is assumed to be small, ψ can be expanded as a sum of even powers of s,

$$\psi=\psi_0+s^2\psi_2+s^4\psi_4+\cdots, \tag{12}$$

where ψ_0, ψ_2, ψ_4 satisfy the following series of equations:

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}-2i\frac{\partial}{\partial\zeta}\right)\psi_0=0, \tag{13}$$

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}-2i\frac{\partial}{\partial\zeta}\right)\psi_2=-s^2\frac{\partial^2}{\partial\zeta^2}\psi_0, \tag{14}$$

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}-2i\frac{\partial}{\partial\zeta}\right)\psi_4=-s^2\frac{\partial^2}{\partial\zeta^2}\psi_2. \tag{15}$$

Equation (18) is the well-known paraxial equation with the fundamental Hermite-Gaussian solution

$$\psi_0=iQe^{-i\rho^2Q}, \tag{16}$$

in which $\rho^2=\xi^2+\eta^2$ and $Q=1/(i+2\zeta)$. By substituting the above equation into Eq. (19), Davis [21] found ψ_2 to be

$$\psi_2=(2iQ+i\rho^4Q^3)\psi_0, \tag{17}$$

and by using this ψ_2, Barton and Alexander [20] found ψ_4 from Eq. (20) to be

$$\psi_4=(-6Q^2-3\rho^4Q^4-2i\rho^6Q^5-0.5\rho^8Q^6)\psi_0. \tag{18}$$

Through our work, we know that ψ_2 and ψ_4 are only two special cases of more general solutions. To show this, we assume ψ_2 to be of the form

$$\psi_2=\phi_2(\rho,Q)\psi_0. \tag{19}$$

Substituting the above equation into Eq. (19) results in

$$\left(\rho\frac{\partial^2}{\partial\rho^2}+\frac{\partial}{\partial\rho}-4iQ\rho^2\frac{\partial}{\partial\rho}+4iQ^2\rho\frac{\partial}{\partial Q}\right)\phi_2=16iQ^3\rho+4Q^4\rho^5-8Q^2\rho. \tag{20}$$

By intensive and somewhat tedious calculations, we can obtain the polynomial solution of ϕ_2 to be

$$\phi_2=C_1Q+(-2-iC_1)Q^2\rho^2+iQ^3\rho^4, \tag{21}$$

where C_1 is an arbitrary constant. The technique used to obtain Eq. (21) is to assume a general polynomial expansion, i.e.,

$$\phi_2=\sum_{m,n=1}^{+\infty}C_{mn}Q^m\rho^n. \tag{22}$$

Then, substituting it into Eq. (20), we can get the solution by equating the coefficients of the factors of the same power on both sides of Eq. (20). From this procedure, we can see that Eq. (21) should be a quite general solution to Eq. (20). The same is true for Eq. (19) and Eq. (14). For example, if there exists another different solution satisfying Eq. (14), which is assumed to be F_2 satisfying Eq. (14), it can be shown that

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}-2i\frac{\partial}{\partial\zeta}\right)(\psi_2-\mathsf{F}_2)=0. \tag{23}$$

Because we are now seeking the Hermite-Gaussian (0,0) mode solution, it is required that

$$\psi_2-\mathsf{F}_2=C\psi_0 \tag{24}$$

where C is an arbitrary constant. This is merely equivalent to adding a trivial constant C to ϕ_2 in solving Eq. (20). So there is no more general solution to Eq. (14) than Eq. (21) as far as the Hermite-Gaussian (0,0) mode field equations are concerned.

Once ψ_2 is known, in the same way, we can continue to find the general solution ψ_4, which can be expressed as

$$\psi_4=[A_1Q^2+(-6C_1-2iA_1)Q^3\rho^2+(6+6iC_1-0.5A_1)Q^4\rho^4+(-4i+C_1)Q^5\rho^6-0.5Q^6\rho^8]\psi_0, \tag{25}$$

in which A_1 is another constant.

However the electromagnetic field components thus obtained from Eq. (19) and Eq. (25) through Eqs. (7) and (8) lack symmetry in the electric and magnetic fields since B_x is always equal to zero while the other components are not. To get symmetric field equations, we start with the Herz vector in the $\hat{e}_y$ direction, namely,

$$\vec{M}'=\psi'(x,y,z)e^{-ikz}\hat{e}_y, \tag{26}$$

$$\vec{A}'=\vec{M}', \tag{27}$$

$$\phi'=\frac{ic}{k}\vec{\nabla}\cdot\vec{M}. \tag{28}$$

Then, repeating the same procedures, we acquire another fifth-order corrected field equation, in which E_y is always equal to zero.

Finally, by superimposing the above two groups of solutions, we can get symmetric electromagnetic field components, which are summarized as follows:

$$E_x=[1+(X_1-2\xi^2Q^2)s^2+(X_2+X_3\xi^2)s^4]\psi_0e^{-i\zeta/s^2}, \tag{29}$$

$$E_y=(-2Q^2s^2+X_3s^4)\xi\eta\psi_0e^{-i\zeta/s^2}, \tag{30}$$

$$E_z=(-2Qs+X_4s^3+X_5s^5)\xi\psi_0e^{-i\zeta/s^2}, \tag{31}$$

$$cB_x=(-2Q^2s^2+X_3s^4)\xi\eta\psi_0e^{-i\zeta/s^2}, \tag{32}$$

$$cB_y=[1+(X_1-2\eta^2Q^2)s^2+(X_2+X_3\eta^2)s^4]\psi_0e^{-i\zeta/s^2}, \tag{33}$$

$$cB_z=(-2Qs+X_4s^3+X_5s^5)\eta\psi_0e^{-i\zeta/s^2}, \tag{34}$$

in which

$$X_1=(C_1-2i)Q+(-3-iC_1)Q^2\rho^2+iQ^3\rho^4, \tag{35}$$

$$X_2=[A_1-2-4iC_1]Q^2+(10i-11C_1-2iA_1)Q^3\rho^2+(13+7iC_1-0.5A_1)Q^4\rho^4+(-5i+C_1)Q^5\rho^6-0.5Q^6\rho^8, \tag{36}$$

$$X_3=(12i-6C_1)Q^3+(12+2iC_1)Q^4\rho^2-2iQ^5\rho^4, \tag{37}$$

$$X_4=(8i-4C_1)Q^2+(10+2iC_1)Q^4\rho^2-2iQ^4\rho^4, \tag{38}$$

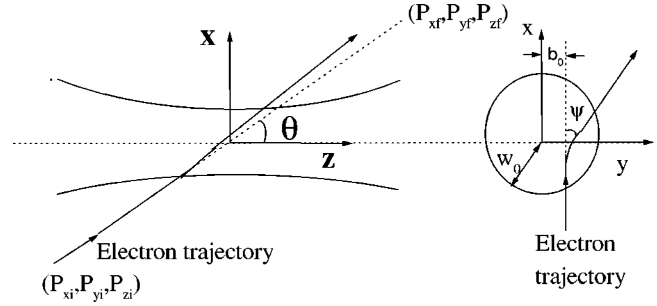

FIG. 1. Schematic configuration of electron scattering by a laser beam. The laser propagates along the z axis. The parameter w_0 is the beam width at the waist. Without losing generality, we assume that the free electrons come in from the negative x side and parallel to the x-z plane. The quantity $(\gamma_i,P_{xi},P_{yi},P_{zi})$ denotes the incoming energy and momentum of the electron and $(\gamma_f,P_{xf},P_{yf}=0,P_{zf})$ that of the outgoing electron. γ is the Lorentz factor and b_0 the impact parameter. $\theta=\tan^{-1}(P_{xi}/P_{zi})$ is the incident angle of the electron, and $\psi=\tan^{-1}(P_{yf}/P_{xf})$ the deflection angle in the x-y plane.

$$X_5=(12+24iC_1-6A_1)Q^3+(-60i+48C_1+6iA_1)Q^4\rho^2+(-54-20iC_1+A_1)Q^5\rho^4+(14i-2C_1)Q^6\rho^6+Q^7\rho^8. \tag{39}$$

As should be expected, when we put $C_1=2i$ and $A_1=-6$, Eqs. (31)–(41) return to the special case obtained by Barton and Alexander [20].

III. ELECTRON DYNAMICS WITH DIFFERENT FIELD PARAMETERS

As found by numerical investigations of the relative percent error by Barton and Alexander [20], the fifth-order corrected field equations can satisfy the Maxwell equations to high accuracy. For example, if a deviation of 1% is acceptable, then the field equations can be used for s less than about 0.2, which means that the beam waist radius to wavelength ratio is about 0.8. But it should be mentioned here that, as far as the electron dynamics in the laser fields are concerned, the absolute error may be a more stringent criterion to decide the applicability of the field equations, especially when very strong lasers are involved. We are not sure whether it is possible to minimizing the errors by optimizing the two arbitrary parameters in our field equations since detailed and comprehensive work is needed in order to check this possibility, which is not our main interest here. In the following, we will study the variations of electron dynamics with C_1 and A_1.

The interaction configuration between the free electron and the laser beam in vacuum is presented in Fig. 1, in which the electron will be injected from the negative x axis side into the laser beam with a small crossing angle to the field propagation direction, the z axis in our case. The electron motion is obtained by solving the relativistic Lorentz-Newtonian equation with a fourth-order Runge-Kutta numerical method, as in our previous work [7,19].

By observing Eqs. (31)–(41), we find that the factors in

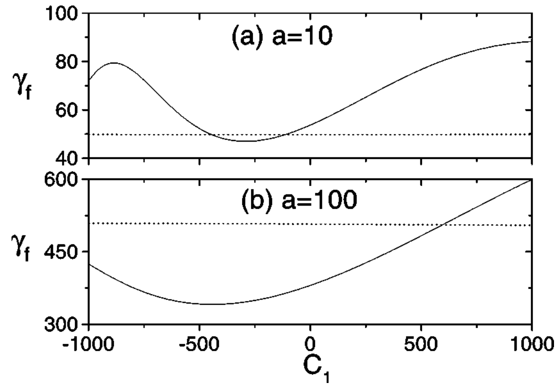

FIG. 2. Parametric dependence of the electron final energy, measured by γ_f in units of $m_e c^2$. In (a), the electron is injected with $P_{xi}/m_e c=5$, $P_{yi}/m_e c=0$, $P_{zi}/m_e c=50$ into the laser beam with $a=10$ and $kw_0=100$ (solid line) and 200 (dotted line), (b) the same but for $P_{xi}/m_e c=50$, $P_{yi}/m_e c=0$, $P_{zi}/m_e c=500$, and $a=100$.

the expansions including A_1 are at least of order s^3, compared with s^2 in the factors including C_1. Thus the influence of A_1 upon the electron dynamics should be much smaller than that of C_1, especially for large beam width. Two examples are presented in Fig. 2 to show the final electron energy variations against the free parameter C_1 under different dimensionless laser intensity $a=eE_0/m_e\omega c=10$ and 100, where $-e$ and m_e are the electron charge and rest mass, respectively, E_0 the reference electric field intensity, ω the laser circular frequency, and c the light speed in vacuum. Just as expected, for larger beam width, e.g., $kw_0=200$ in the figure, the electron final energies change very little. But when kw_0 is reduced to 100, we observe nearly 50% energy variation in the range $-1000\leqslant C_1\leqslant 1000$. It should be mentioned that C_1 cannot be made arbitrarily large since in order to keep the field expansion valid it is necessary to have $C_1 s^2 \ll 1$. It appears that the importance of the first-order corrections, which directly result in the discovery of longitudinal field components, has been well established now, results that further demonstrate the importance of the higher-order corrections to the laser fields in the description of the electron dynamics. The great care that should be taken in the laser field description has been expounded in detail by Hora *et al.* in a recent paper [11], which focuses on the principle of high accuracy for the nonlinear theory of electron acceleration in vacuum. Another interesting phenomenon in Fig. 2 is that the parametric dependence of the electron dynamics is much more influenced by the width than by the laser intensity since by increasing a from 10 to 100 the relative energy variations in Figs. 2(a) and 2(b) do not change very much. This implies that this kind of parametric dependence can be well studied in the low laser intensity region ($a\sim 10$). This can be understood when the following fact is considered: a appears linearly in the field equations while the factors related to C_1 depend upon the beam width in the form of $s^2 =(1/kw_0)^2$, s^3, etc. In our calculations, we have not found much parametric dependence of A_1 using the same parameters as in Fig. 2. This tells us that the second-order corrections should play the predominant role in the above effect since A_1 only starts to appear in the third-order corrections.

IV. FIFTH-ORDER CORRECTED FIELD EQUATIONS FOR THE HERMITE-GAUSSIAN (0,1) MODE LASER BEAM

For laser beams of Hermite-Gaussian (0,1) mode, the procedures to obtain the field equations are the same as in Sec. II except that

$$\psi_0=Q^2\xi e^{-i\rho^2 Q}, \tag{40}$$

from which we obtain

$$\psi_2=[C_1'Q+(-3-0.5iC_1')Q^2\rho^2+iQ^3\rho^4]Q^2\xi e^{-i\rho^2 Q}, \tag{41}$$

$$\begin{aligned}\psi_4=\Big[&A_1'Q^2+(-iA_1'-6C_1')Q^3\rho^2\\&+\left(-\frac{1}{6}A_1'+4iC_1'+10\right)Q^4\rho^4\\&+(-5i+0.5C_1')Q^5\rho^6-0.5Q^6\rho^8\Big]Q^2\xi e^{-i\rho^2 Q}.\end{aligned} \tag{42}$$

Here C_1 and A_1 are two arbitrary constants. After intensive calculations, we summarize the corresponding field equations as follows.

First, for the Hertz vector polarized along $\hat{e}_x$, i.e.,

$$\vec{M}=(\psi_0+\psi_2 s^2+\psi_4 s^4)e^{-ikz}\hat{e}_x, \tag{43}$$

we have

$$E_x=[1+Y_1 s^2+(Y_2+Y_3\xi^2)s^4]\xi Q^2 e^{-i\rho^2 Q-i\zeta/s^2}, \tag{44}$$

$$E_y=[(-2iQ-4Q^2\xi^2)s^2+(Y_4+Y_5\xi^2)s^4]\eta Q^2 e^{-i\rho^2 Q-i\zeta/s^2}, \tag{45}$$

$$\begin{aligned}E_z=[&(-i+2Q\xi^2)s+(Y_6+Y_7\xi^2)s^3\\&+(Y_8+Y_9\xi^2)s^5]Q^2 e^{-i\rho^2 Q-i\zeta/s^2},\end{aligned} \tag{46}$$

$$B_x=0, \tag{47}$$

$$B_y=(1+Y_{10}s^2+Y_{11}s^4)\xi Q^2 e^{-i\rho^2 Q-i\zeta/s^2}, \tag{48}$$

$$B_z=(-2Qs+Y_{12}s^3+Y_{13}s^5)\eta\xi Q^2 e^{-i\rho^2 Q-i\zeta/s^2}, \tag{49}$$

in which

$$Y_1=(-6i+C_1')Q-4\xi^2 Q^2+(-3-0.5iC_1')Q^2\rho^2+i\rho^4 Q^3, \tag{50}$$

$$\begin{aligned}Y_2=&A_1'Q^2+(-18-9iC_1')Q^2+(30i-iA_1'-9C_1')Q^3\rho^2\\&+\left(\frac{1}{6}A_1'+4iC_1'+16\right)Q^4\rho^4+(-5i+0.5C_1')Q^5\rho^6\\&-0.5Q^6\rho^8,\end{aligned} \tag{51}$$

$$Y_3=(32i-8C_1')Q^3+(28+2iC_1')Q^4\rho^2-4iQ^5\rho^4, \tag{52}$$

$$Y_4=(-6-3iC_1')Q^2+(10i-C_1')Q^3\rho^2+2Q^4\rho^4, \tag{53}$$

$$Y_5=(32i-8C_1')Q^3+(28+2iC_1')Q^4\rho^2-4iQ^5\rho^4, \tag{54}$$

$$Y_6=(-4-iC_1')Q^2+(5i-0.5C_1')Q^2\rho^2+Q^3\rho^4, \tag{55}$$

$$Y_7=(18i-3C_1')Q^2+(14+iC_1')Q^3\rho^2-2iQ^4\rho^4, \tag{56}$$

$$\begin{aligned}Y_8=&(-iA_1'-6C_1')Q^2+(-A_1'+12iC_1'+24)Q^3\rho^2\\&+\left(\frac{1}{6}iA_1'+5C_1'-26i\right)Q^4\rho^4+(-7-0.5iC_1')Q^5\rho^6\\&+0.5Q^4\rho^8,\end{aligned} \tag{57}$$

$$\begin{aligned}Y_9=&(-4A_1'+36iC_1'+48)Q^3+\left(\frac{8}{3}iA_1'+44C_1'-152i\right)Q^4\rho^2\\&+\left(\frac{1}{3}A_1'-13iC_1'-94\right)Q^5\rho^4+(18i-C_1')Q^6\rho^6+Q^7\rho^8,\end{aligned} \tag{58}$$

$$Y_{10}=(C_1'-4i)Q+(-5-0.5iC_1')Q^2\rho^2+iQ^3\rho^4, \tag{59}$$

$$\begin{aligned}Y_{11}=&(A_1'-6iC_1')Q^2+(-iA_1'-12C_1'+24i)Q^3\rho^2\\&+\left(-\frac{1}{6}A_1'+5iC_1'+26\right)Q^4\rho^4+(-7i+0.5C_1')Q^5\rho^6\\&-0.5Q^6\rho^8,\end{aligned} \tag{60}$$

$$Y_{12}=(6i-3C_1')Q^2+(10+iC_1')Q^3\rho^2-2iQ^4\rho^4, \tag{61}$$

$$\begin{aligned}Y_{13}=&(-4A_1'+12iC_1')Q^3+\left(\frac{8}{3}iA_1'+28C_1'-40i\right)Q^4\rho^2\\&+\left(\frac{1}{3}A_1'-11iC_1'-50\right)Q^5\rho^4+(14i-C_1')Q^6\rho^6+Q^7\rho^8.\end{aligned} \tag{62}$$

Second, when the Hertz vector is polarized along $\hat{e}_y$, for symmetry reasons, we use

$$\psi_0'=Q^2\eta e^{-i\rho^2 Q}, \tag{63}$$

$$\psi_2'=[C_1'Q+(-3-0.5iC_1')Q^2\rho^2+iQ^3\rho^4]Q^2\eta e^{-i\rho^2 Q}, \tag{64}$$

$$\begin{aligned}\psi_4'=&\Bigg[A_1'Q^2+(-iA_1'-6C_1')Q^3\rho^2\\&+\left(-\frac{1}{6}A_1'+4iC_1'+10\right)Q^4\rho^4\\&+(-5i+0.5C_1')Q^5\rho^6-0.5Q^6\rho^8\Bigg]Q^2\eta e^{-i\rho^2 Q}\end{aligned} \tag{65}$$

to replace ψ_0, ψ_2, and ψ_4 in Eq. (45). Then the related field equations can be obtained as

$$\begin{aligned}E_x=&[(-2iQ-4Q^2\eta^2)s^2\\&+(Y_4+Y_5\eta^2)s^4]\xi Q^2 e^{-i\rho^2 Q-i\zeta/s^2},\end{aligned} \tag{66}$$

$$E_y=[1+Y_1 s^2+(Y_2+Y_3\eta^2)s^4]\eta Q^2 e^{-i\rho^2 Q-i\zeta/s^2}, \tag{67}$$

$$\begin{aligned}E_z=&[(-i+2Q\eta^2)s+Y_7\eta^2 s^3\\&+(Y_8+Y_9\eta^2)s^5]Q^2 e^{-i\rho^2 Q-i\zeta/s^2},\end{aligned} \tag{68}$$

$$B_x=(1+Y_{10}s^2+Y_{11}s^4)\eta Q^2 e^{-i\rho^2 Q-i\zeta/s^2}, \tag{69}$$

$$B_y=0, \tag{70}$$

$$B_z=[-2Qs+Y_{12}s^3+Y_{13}s^5]\xi\eta Q^2 e^{-i\rho^2 Q-i\zeta/s^2}. \tag{71}$$

Finally the symmetric field descriptions can be obtained by superimposing the above two groups of equations.

V. SUMMARY

In this paper, we have extended the work of Barton and Alexander [20] on the fifth-order corrected field expressions for a Hermite-Gaussian (0,0) mode laser beam to more general cases with adjustable parameters C_1 and A_1. C_1 begins to appear in the expansion from the factor of order s^2 and A_1 from the factor of order s^3. The parametric dependence of the electron dynamics was investigated by numerical methods. It was found that such dependence is mainly influenced by the beam width and comes from the second-order corrections in the expansions. Finally, the fifth-order corrected field equations for Hermite-Gaussian (0,1) mode were also presented. All these results will be of potential interest in exploring the electron dynamics in strong laser fields, where highly precise analytical descriptions are in great demand since they can greatly simplify the numerical calculations and make the analysis of results much more efficient. To study the physical consequences of all these high-order field expressions in laser acceleration will be our next work.

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